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# The Anthropic Landscape of String Theory

*L. Susskind*

Department of Physics  
Stanford University  
Stanford, CA 94305-4060

## Abstract

In this lecture I make some educated guesses, about the landscape of string theory vacua. Based on the recent work of a number of authors, it seems plausible that the landscape is unimaginably large and diverse. Whether we like it or not, this is the kind of behavior that gives credence to the Anthropic Principle. I discuss the theoretical and conceptual issues that arise in developing a cosmology based on the diversity of environments implicit in string theory.

# 1 The Landscape

The world-view shared by most physicists is that the laws of nature are uniquely described by some special action principle that completely determines the vacuum, the spectrum of elementary particles, the forces and the symmetries. Experience with quantum electrodynamics and quantum chromodynamics suggests a world with a small number of parameters and a unique ground state. For the most part, string theorists bought into this paradigm. At first it was hoped that string theory would be unique and explain the various parameters that quantum field theory left unexplained. When this turned out to be false, the belief developed that there were exactly five string theories with names like type-2a and Heterotic. This also turned out to be wrong. Instead, a continuum of theories were discovered that smoothly interpolated between the five and also included a theory called M-Theory. The language changed a little. One no longer spoke of different theories, but rather different solutions of some master theory. The space of these solutions is called *The Moduli Space of Supersymmetric Vacua*. I will call it the supermoduli-space. Moving around on this supermoduli-space is accomplished by varying certain dynamical *moduli*. Examples of moduli are the size and shape parameters of the compact internal space that 4-dimensional string theory always needs. These moduli are not parameters in the theory but are more like fields. As you move around in ordinary space, the moduli can vary and have their own equations of motion. In a low energy approximation the moduli appear as massless scalar fields. The beauty of the supermoduli-space point of view is that there is only one theory but many solutions which are characterized by the values of the scalar field moduli. The mathematics of the string theory is so precise that it is hard to believe that there isn't a consistent mathematical framework underlying the supermoduli-space vacua.

However the continuum of solutions in the supermoduli-space are all supersymmetric with exact super-particle degeneracy and vanishing cosmological constant. Furthermore they all have massless scalar particles, the moduli themselves. Obviously none of these vacua can possibly be our world. Therefore the string theorist must believe that there are other discrete islands lying off the coast of the supermoduli-space. The hope now is that a single non-supersymmetric island or at most a small number of islands exist and that non-supersymmetric physics will prove to be approximately unique. This view is not inconsistent with present knowledge (indeed it is possible that there are no such islands) but I find it completely implausible. It is much more likely that the number of

discrete vacua is astronomical, measured not in the millions or billions but in googles or googleplexes <sup>1</sup>.

This change in viewpoint is demanded by two facts, one observational and one theoretical. The first is that the expansion of the universe is accelerating. The simplest explanation is a small but non-zero cosmological constant. Evidently we have to expand our thinking about vacua to include states with non-zero vacuum energy. The incredible smallness and apparent fine tuning of the cosmological constant makes it absurdly improbable to find a vacuum in the observed range unless there are an enormous number of solutions with almost every possible value of  $\lambda$ . It seems to me inevitable that if we find one such vacuum we will find a huge number of them. I will from now on call the space of all such string theory vacua the *landscape*.

The second fact is that some recent progress has been made in exploring the landscape [2, 7]. Before explaining the new ideas I need to define more completely what I mean by the landscape. The supermoduli-space is parameterized by the moduli which we can think of as a collection of scalar fields  $\Phi_n$ . Unlike the case of Goldstone bosons, points in the moduli space are not related by a symmetry of the theory. Generically, in a quantum field theory, changing the value of a non-Goldstone scalar involves a change of potential energy. In other words there is a non-zero field potential  $V(\Phi)$ . Local minima of  $V$  are what we call vacua. If the local minimum is an absolute minimum the vacuum is stable. Otherwise it is only metastable. The value of the potential energy at the minimum is the cosmological constant for that vacuum.

To the extent that the low energy properties of string theory can be approximated by field theory, similar ideas apply. Bearing in mind that the low energy approximation may break down in some regions of the landscape, I will assume the existence of a set of fields and a potential. The space of these fields is the landscape.

The supermoduli-space is a special part of the landscape where the vacua are supersymmetric and the potential  $V(\Phi)$  is exactly zero. These vacua are marginally stable and can be excited by giving the moduli arbitrarily small time derivatives. On the supermoduli-space the cosmological constant is also exactly zero. Roughly speaking, the supermoduli-space is a perfectly flat plain at exactly zero altitude <sup>2</sup>. Once we move off the plain, supersymmetry is broken and a non-zero potential develops, usually through

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<sup>1</sup>A google is defined to be ten to the power one hundred. That is  $\mathcal{G} = 10^{100}$ . A googleplex is  $10^{\mathcal{G}}$

<sup>2</sup>By altitude I am of course referring to the value of  $V$ .

some non-perturbative mechanism. Thus beyond the flat plain we encounter hills and valleys. We are particularly interested in the valleys where we find local minima of  $V$ . Each such minimum has its own vacuum energy. The typical value of the potential difference between neighboring valleys will be some fraction of  $M_p^4$  where  $M_p$  is the Planck mass. The potential barriers between minima will also be of similar height. Thus if a vacuum is found with cosmological constant of order  $10^{-120}M_p^4$ , it will be surrounded by much higher hills and other valleys.

Next consider two large regions of space, each of which has the scalars in some local minimum, the two minima being different. If the local minima are landscape-neighbors then the two regions of space will be separated by a domain wall. Inside the domain wall the scalars go over a “mountain pass”. The interior of the regions are vacuum like with cosmological constants. The domain wall which can also be called a membrane has additional energy in the form of a membrane tension. Thus there will be configurations of string theory which are not globally described by a single vacuum but instead consists of many domains separated by domain walls. Accordingly, the landscape in field space is reflected in a complicated terrain in real space.

There are scalar fields that are not usually thought of as moduli but once we leave the flat plain I don’t think there is any fundamental difference. These are the four-form field strengths first introduced in the context of the cosmological constant by Brown and Teitelboim [1]. A simple analogy exists to help visualize these fields and their potential. Think of 1+1 dimensional electrodynamics with electric fields<sup>3</sup>  $E$  and massive electrons. The electric field is constant in any region of space where there are no charges. The field energy is proportional to the square of the field strength. The electric field jumps by a quantized unit whenever an electron is passed. Going in one direction, say along the positive  $x$  axis, the field makes a positive unit jump when an electron is passed and a negative jump when a positron is passed. In this model different vacua are represented by different quantized values of the electric field while the electrons/positrons are the domain walls. The energy of a vacuum is proportional to  $E^2$ . This model is not fundamentally different than the case with scalar fields and a potential. In fact by bosonizing the theory it can be expressed as a scalar field theory with a potential

$$V(\phi) = c\phi^2 + \mu \cos\phi. \tag{1.1}$$

If  $\mu$  is not too small there are many minima representing the different possible 2-form field

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<sup>3</sup>In 1+1 dimensions there is no magnetic field and the electric field is a two form, aka, a scalar density.

strengths, each with a different energy.

In 3+1 dimensions the corresponding construction requires a 4-form field strength  $F$  whose energy is also proportional to  $F^2$ . This energy appears in the gravitational field equations as a positive contribution to the cosmological constant. The analogue of the charged electrons are membranes which appear in string theory and function as domain walls to separate vacua with different  $F$ . This theory can also be written in terms of a scalar field with a potential similar to 1.1. In the future I will include such fields along with moduli as coordinates of the landscape.

Let's now consider a typical compactification of M-theory from eleven to 4 dimensions. The simplest example is gotten by choosing for the compact directions a 7-torus. The torus has a number of moduli representing the sizes and angles between the seven 1-cycles. The 4-form fields have as their origin a 7-form field strength <sup>4</sup> which is one of the fundamental fields of M-theory. The 7-form fields have 7 anti-symmetrized indecies. These non-vanishing 7-form can be configured so that three of the indecies are identified with compact dimensions and the remainder with uncompactified spacetime. This can be done in thirty five  $= (7 \times 6 \times 5)/(1 \times 2 \times 3)$  ways which means that there are that many distinct 4-form fields in the uncompactified non-compact space. More generally, in the kinds of compact manifolds used in string theory to try to reproduce standard model physics there can be hundreds of independent ways of "wrapping" three compact directions with flux, thus producing hundreds of 3+1 dimensional 4-form fields. As in the case of 1+1 dimensional quantum electrodynamics, the field strengths are quantized, each in integer multiples of a basic field unit. A vacuum is specified by a set of integers  $n_1, n_2, \dots, n_N$  where  $N$  can be as big as several hundred or more. The energy density of the energy of the 4-form fields has the form

$$\epsilon = \sum_{i=1}^N c_i N_i^2 \quad (1.2)$$

where the constants  $c_i$  depend on the details of the compact space.

The analogue of the electrons and positrons of the 1+1 dimensional example are branes. The 11 dimensional M-theory has 5-branes which fill 5 spatial directions and time. By wrapping 5-branes the same way the fluxes of the 4-forms are wrapped on internal 3-cycles leaves 2-dimensional membranes in 3+1 dimensions. These are the domain walls

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<sup>4</sup>The fact that we have the number 7 appearing in two ways, as the number of compact dimensions and as the number of indecies of the field strength is accidental.

which separate different values of field strength. There are  $N$  types of domain wall, each allowing a unit jump of one of the 4-forms.

Bousso and Polchinski [2] begin by assuming they have located some deep minimum of the field potential at some point  $\Phi_0$ . The value of the potential is supposed to be very negative at this point, corresponding to a negative cosmological constant,  $\lambda_0$  of order the Planck scale. Also the 4-forms are assumed to vanish at this point. They then ask what kind of vacua can they obtain by discretely increasing the 4-forms. The answer depends to some degree on the compactification radii on the internal space but with modest parameters it is not hard to get such a huge number of vacua that it is statistically likely to have one in the range  $\lambda \sim 10^{-120} M_p^4$ .

To see how this works we write the cosmological constant as the sum of two terms, is the cosmological constant for vanishing 4-form, and the contribution of the 4-forms,

$$\lambda = \lambda_0 + \sum_{i=1}^N c_i N_i^2. \quad (1.3)$$

With a hundred terms and modestly small values for the  $c_n$  it is highly likely to find a value of  $\lambda$  in the observed range. Note that no fine tuning is required, only a very large number of ways to make the vacuum energy.

The problem with [2] was clearly recognized by the authors; The starting point is so far from the supermoduli-space that none of the usual tools of approximate supersymmetry are available to control the approximation. The example was intended only as a model of what might happen because of the large number of possibilities.

More recently Kachru, Kallosh, Linde and Trivedi [7] have improved the situation by finding an example which is more under control. These authors subtly use the various ingredients of string theory including fluxes, branes, anti-branes and instantons to construct a rather tractable example with a small positive cosmological constant.

In addition to arguing that string theory does have many vacua with positive cosmological constant the argument in [7] tends to dispel the idea that vacua, not on the supermoduli-space, must have vanishing cosmological constant. In other words there is no evidence in string theory that a hoped for but unknown mechanism will automatically force the cosmological constant to zero. It seems very likely that all of the non-supersymmetric vacua have finite  $\lambda$ .

The vacua in [7] are not at all simple. They are jury-rigged, Rube Goldberg contraptions that could hardly have fundamental significance. But in an anthropic theory simplicity and elegance are not considerations. The only criteria for choosing a vacuum

is utility, i.e. does it have the necessary elements such as galaxy formation and complex chemistry that are needed for life. That together with a cosmology that guarantees a high probability that at least one large patch of space will form with that vacuum structure is all we need.

## 2 The Trouble with de Sitter Space

The classical vacuum solution of Einstein’s equations with a positive cosmological constant is de Sitter space. It is doubtful that it has a precise meaning in a quantum theory such as string theory [9, 10, 11]. I want to review some of the reasons for thinking that de Sitter Space is at best a metastable state.

It is important to recognize that there are two very different ways to think about de Sitter Space. The first is to take a global view of the spacetime. The global geometry is described by the metric

$$ds^2 = R^2 \left\{ dt^2 - (\cosh t)^2 d^2\Omega_3 \right\} \quad (2.4)$$

where  $d^2\Omega_d$  is the metric for a unit d–sphere and  $R$  is related to the cosmological constant by

$$R = (\lambda G)^{-\frac{1}{2}}. \quad (2.5)$$

Viewing de Sitter Space globally would make sense if it were a system that could be studied from the outside by a “meta–observer”. Naively, the meta–observer would make use of a (time dependent) Hamiltonian to evolve the system from one time to another. An alternate description would use a Wheeler de Witt formalism to define a wave function of the universe on global space–like slices.

The other way of describing the space is the *causal patch* or “Hot Tin Can” description. The relevant metric is

$$ds^2 = R^2 \left\{ (1 - r^2) dt^2 - \frac{1}{(1 - r^2)} dr^2 - r^2 d^2\Omega_2 \right\} \quad (2.6)$$

In this form the metric is static and has a form similar to that of a black hole. In fact the geometry has a horizon at  $r = 1$ . The static patch does not cover the entire global de Sitter Space but is analogous to the region outside a black hole horizon. It is the region which can receive signals from, and send signals to, an observer located at  $r = 0$ . To such an observer de Sitter Space appears to be a spherical cavity bounded by a horizon a finite distance away.

Experience with black holes has taught us to be very wary of global descriptions when horizons are involved. In a black hole geometry there is no global conventional quantum description of both sides of the horizon. This suggests that a conventional quantum description of de Sitter Space only makes sense within a given observer's causal patch. The descriptions in different causal patches are complementary [3, 4] but can not be put together into a global description without somehow modifying the rules of quantum mechanics.

As in the black hole case, a horizon implies a thermal behavior with a temperature and an entropy. These are given by

$$\begin{aligned} T &= \frac{1}{2\pi R} \\ S &= \frac{\pi R^2}{G}. \end{aligned} \tag{2.7}$$

For the rest of this section I will be assuming the causal patch description of some particular observer.

If the observed “dark energy” in the universe really is a small positive cosmological constant the ultimate future of our universe will be eternal de Sitter Space. This would mean not that the future is totally empty space but that the world will have all the features of an isolated finite thermal cavity with finite temperature and entropy. Thermal equilibrium for such a system is not completely featureless. On short time scales not much can be expected to happen but on very long time scales everything happens. A famous example involves a gas of molecules in a sealed room. Imagine that we start all the molecules in one corner of the room. In a relatively short time the gas will spread out to fill the room and come to thermal equilibrium. During the approach to equilibrium interesting dissipative structures such as droplets, eddies and vortices form and then dissipate. The usual assumption is that nothing happens after that. The entropy has reached its maximum value and the second law forbids any further interesting history. But on a sufficiently long time scale, large fluctuations will occur. In fact the phase point will return over and over to the neighborhood of any point in phase space including the original starting point. These *Poincare recurrences* generally occur on a time scale exponentially large in the thermal entropy of the system. Thus we define the Poincare recurrence time

$$T_r = \exp S. \tag{2.8}$$

On such long time scale the second law of thermodynamics will repeatedly be violated by large scale fluctuations.



Thus even a pure de Sitter Space would have an interesting cosmology of sorts. The causal patch of any observer would undergo Poincare recurrences in which it would endlessly fluctuate back to a state similar to its starting point, but each time slightly different.

The trouble with such a cosmology is that it relies on very rare “miracles” to start it off each time. But there are other miracles which could occur and lead to anthropically acceptable worlds with a vastly larger probability than our world. Roughly speaking the relative probability of a fluctuation leading to a given configuration is proportional to the exponential of its entropy. An example of a configuration far more likely than our own would be a world in which everything would be just like our universe except the temperature of the cosmic microwave background was ten degrees instead of three. When I say everything is the same I am including such details as the abundance of the elements.

Ordinarily such a universe would be ruled out on the grounds that it would take a huge miracle for the helium and deuterium to survive the bombardment by the extra photons implied by the higher temperature. That is correct, a fantastic miracle would be required, but such miracles would occur far more frequently than the ultimate miracle of returning to the starting point. This can be argued just from the fact that a universe at 10 degrees K has a good deal more entropy than one at 3 degrees. In a world based on recurrences it would be overwhelming unlikely that cosmology could be traced back to something like the inflationary era without a miraculous reversal of the second law along the way. Thus we are forced to conclude that the sealed tin can model of the universe must be incorrect, at least for time scales as long as the recurrence time.

Another difficulty with an eternal de Sitter Space involves a mathematical conflict between the symmetry of de Sitter Space and the finiteness of the entropy [9]. Basically the argument is that the finiteness of the de Sitter Space entropy indicates that the spectrum of energy is discrete. It is possible to prove that the symmetry algebra of de Sitter Space can not be realized in a way which is consistent with the discreteness of this spectrum. In fact this problem is not independent of the issues of recurrences. The discreteness of the spectrum means that there is a typical energy spacing of order

$$\Delta E \sim \exp -S. \quad (2.9)$$

The discreteness of the spectrum can only manifest itself on time scales of order  $(\Delta E)^{-1}$  which is just the recurrence time. Thus there are problems with realizing the full symmetries of de Sitter Space for times as long as  $T_r$ .

Finally another difficulty for eternal de Sitter Space is that it does not fit at all well

with string theory. Generally the only objects in string theory which are rigorously defined are S-Matrix elements. Such an S matrix can not exist in a thermal background. Part of the problem is again the recurrences which undermine the existence of asymptotic states. Unfortunately there are no known observables in de Sitter Space which can substitute for S-matrix elements. The unavoidable implication of the issues I have raised is that eternal de Sitter Space is an impossibility in a properly defined quantum theory of gravity.

### 3 de Sitter Space is Unstable

In [7] a particular string theory vacuum with positive  $\lambda$  was studied. One of the many interesting things that the authors found was that the vacuum is unstable with respect to tunneling to other vacua. In particular the vacuum can tunnel back to the supermoduli-space with vanishing cosmological constant. Using instanton methods the authors calculated that the lifetime of the vacuum is less than the Poincare recurrence time. This is no accident. To see why it always must be so, let's consider the effective potential that the authors of [7] derived. The only modulus which is relevant is the overall size of the compact manifold  $\Phi$ . The potential is shown in Figure 1. The de Sitter vacuum occurs at the point  $\Phi = \Phi_0$ . However, the absolute minimum of the potential occurs not at  $\Phi_0$  but at  $\Phi = \infty$ . At this point the vacuum energy is exactly zero and the vacuum one of the ten dimensional vacua of the supermoduli-space. As was noted long ago by Dine and Seiberg there are always runaway solutions like this in string theory. The potential on the supermoduli-space is zero and so it is always possible to lower the energy by tunneling to a point on the supermoduli-space.

Suppose we are stuck in the potential well at  $\Phi_0$ . The vacuum of the causal patch has a finite entropy and fluctuates up and down the walls of the potential. One might think that fluctuations up the sides of the potential are Boltzmann suppressed. In a usual thermal system there are two things that suppress fluctuations. The first is the Boltzmann suppression by factor

$$\exp -\beta E$$

and the second is entropy suppression by factor

$$\exp S_f - S$$

where  $S$  is the thermal entropy and  $S_f$  is the entropy characterizing the fluctuation which is generally smaller than  $S$ . However in a gravitational theory in which space is bounded

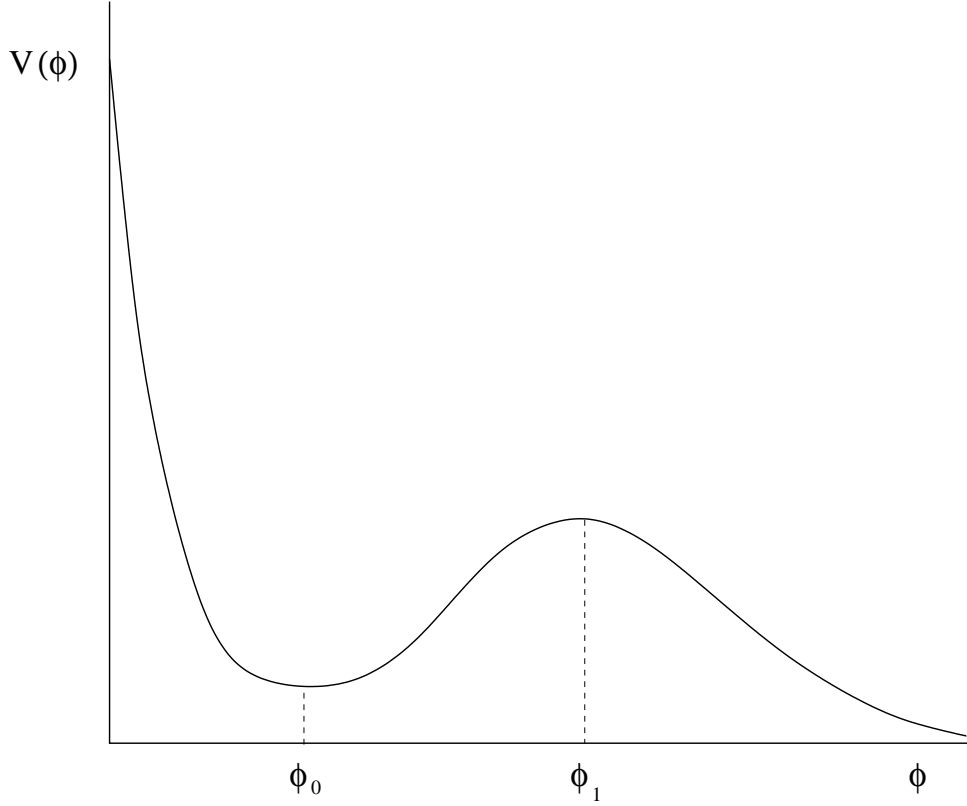


Figure 1:

(as in the static patch) the total energy is always zero, at least classically. Hence the only suppression is entropic. The phase point wanders around in phase space spending a time in each region proportional to its phase space volume, i.e.  $\exp -S_f$ . Furthermore the typical time scale for such a fluctuation to take place is of order

$$T_f \sim \exp S - S_f. \quad (3.1)$$

Now consider a fluctuation which brings the field  $\phi$  to the top of the local maximum at  $\phi = \phi_1$  in the entire causal patch. The entropy at the top of the potential is given in terms of the cosmological constant at the top. It is obviously positive and less than the entropy at  $\phi_0$ . Thus the time for the field to fluctuate to  $\phi_1$  (over the whole causal patch) is strictly less than the recurrence time  $\exp S$ . But once the field gets to the top there is no obstruction to it rolling down the other side to infinity. It follows that a de Sitter vacuum of string theory is never longer lived than  $T_r$  and furthermore we end up at a supersymmetric point of vanishing cosmological constant.

There are other possibilities. If the cosmological constant is not very small it may tunnel over the nearest mountain pass to a neighboring valley of smaller positive cosmological constant. This will also take place on a time scale which is too short to allow recurrences. By the same argument it will not stay in the new vacuum indefinitely. It may find a vacuum with yet smaller cosmological constant to tunnel to. Eventually it will have to make a transition out of the space of vacua with positive cosmological constants<sup>5</sup>.

## 4 Bubble Cosmology

To make use of the enormous diversity of environments that string theory is likely to bring with it, we need a dynamical cosmology which, with high probability, will populate one or more regions of space with an anthropically favorable vacuum. There is a natural candidate for such a cosmology that I'll explain from the global perspective.

For simplicity let's temporarily assume that there are only two vacua, one with positive cosmological constant  $\lambda$ , and one with vanishing cosmological constant. Without worrying how it happened we suppose that some region of the universe has fallen into the minimum with positive cosmological constant. From the global perspective it is inflating and new Hubble volumes are constantly being produced by the expansion. Pick a time-like observer who looks around and sees a static universe bounded by a horizon. The observer will eventually observe a transition in which his entire observable region slides over the mountain pass and settles to the region of vanishing  $\lambda$ . The observer sees the horizon-boundary quickly recede, leaving in its wake an infinite open Freedman, Robertson, Walker universe with negative spatial curvature. The final geometry has light-like and time like future infinities similar to flat space.

It is helpful to draw some Penrose diagrams to illustrate the history. For this purpose we turn to the global point of view. First draw a diagram representing pure de Sitter Space. See Figure 2. The figure also shows two observers whose causal patches overlap for some period of time. In Figure 3, the same geometry is shown except that the formation of a bubble of  $\lambda = 0$  vacuum is also depicted. The bubble is created at point ( $a$ ) and expands with velocity that approaches the speed of light. Eventually the growing bubble intersects

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<sup>5</sup>Tunneling to vacua with negative cosmological constant may or may not be a possibility. However such a transition will eventually lead to a crunch-singularity. Whether the system survives the crunch is not known. It should be noted that transitions to negative cosmological constant are suppressed and can even be forbidden depending on magnitudes of the vacuum energies, and the domain wall tension. I will assume that such transitions do not occur.

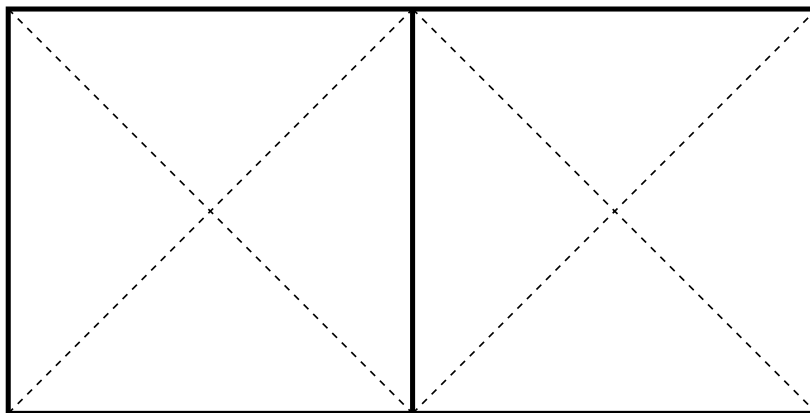


Figure 2:

the infinite future of the de Sitter Space but the geometry inside the bubble continues and forms a future infinite null. Notice that even though the observer's final universe is infinite his past light cone does not include the whole global space-time. In fact his causal diamond is not much bigger than it would have been if the space never decayed.

The region outside the bubble is still inflating and disappearing out of causal contact with the observer. From the causal patch viewpoint the entire world has been swallowed by the bubble.

Now let us take the more global view. The bubble does not swallow the entire global space but leaves part of the space still inflating. Inevitably bubbles will form in this region. In fact if we follow the world line of any observer, it will eventually be swallowed by a bubble of  $\lambda = 0$  vacuum. The line representing the remote future in Figure 3 is replaced by a jagged fractal as in Figure 4. Any observer eventually ends up at the top of the diagram in one of infinitely many time-like infinities. This process leading to infinitely many disconnected bubble universes is essentially similar to the process of eternal inflation envisioned by Linde.

The real landscape is not comprised of only two vacua. If an observer starts with a large value of the cosmological constant there will be many ways for the causal patch to descend to the supermoduli-space. From the global viewpoint a bubbles will form in neighboring valleys with somewhat smaller cosmological constant. Since each bubble has a positive cosmological constant it will be inflating but the space between bubbles is inflating faster so the bubbles go out of causal contact with one another. Each bubble evolves in

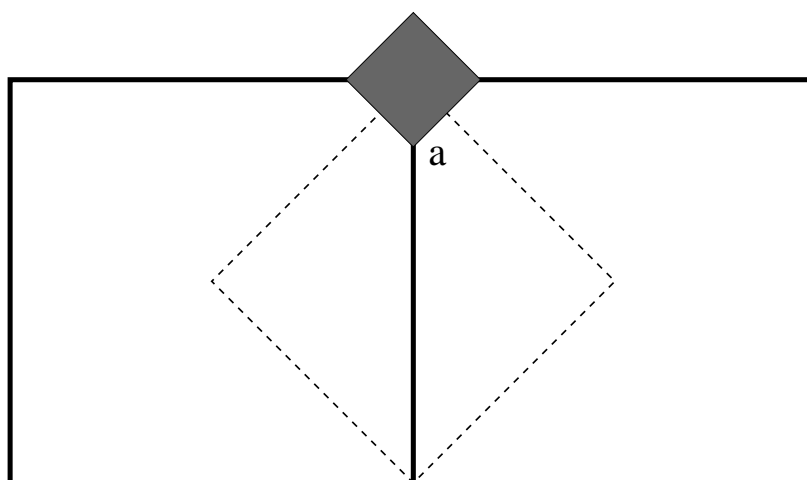


Figure 3:

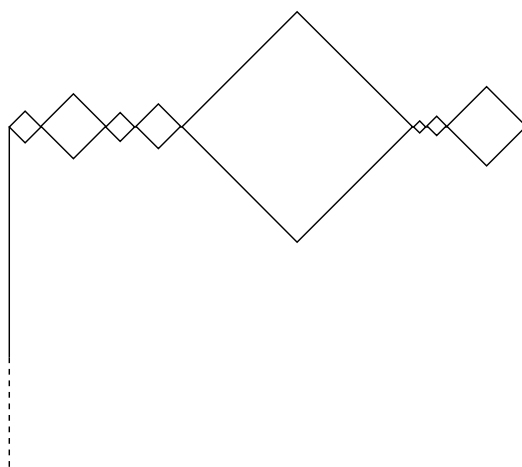


Figure 4:

isolation from all the others. Furthermore, in a time too short for recurrences, bubbles will nucleate within the bubbles. Following a single observer within his own causal patch, the cosmological constant decreases in a series of events until the causal patch finds itself in the supermoduli-space. Each observer will see a series of vacuums decending down to the supermoduli-space and the chances that he passes through an anthropically acceptable vacuum is most likely very small. But on the other hand the global space contains an infinite number of such histories and some of them will be acceptable.

The only problem with the cosmology that I just outlined is that it is formulated in global coordinates. From the viewpoint of any causal patch, all but one of the bubbles is outside the horizon. As I've emphasized, the application of the ordinary rules of quantum mechanics only makes sense within the horizon of an observer. We don't know the rules for putting together the various patches into one comprehensive global description and until we do there can not be any firm basis for the kind of anthropic cosmology I described. Nevertheless the picture is tempting.

## 5 Cosmology as a Resonance

The idea of scalar fields and potentials is approximate once we leave the supermoduli-space. So is the notion of a stable de Sitter vacuum. The problem is familiar. How do we make precise sense of an unstable state in quantum mechanics. In ordinary quantum mechanics the clearest situation is when we can think of the unstable state as a resonance in a set of scattering amplitudes. The parameters of a resonance, i.e. its width and mass are well defined and don't depend on the exact way the resonance was formed. Thus even black holes have precise meaning as resonant poles in the S-matrix. Normally we can not compute the scattering amplitudes that describe the formation and evaporation of a black hole but it is comforting that an exact criterion exists.

In the case of a black hole the density of levels is enormous being proportional to the exponential of the entropy. The spacing between levels is therefore exponentially small. On the other hand the width of each level is not very small. The lifetime of a state is the time that it takes to emit a single quantum of radiation and this is proportional to the Schwarzschild radius. Therefore the levels are broadened by much more than their spacing. The usual resonance formulas are not applicable but the precise definition of the unstable state as a pole in the scattering amplitude is. I think the same things can be said about the unstable de Sitter vacua but it can only be understood by returning to the

causal patch way of thinking. Therefore let's focus on the causal patch of one observer. We have discussed the observer's future history and found that it always ends in an infinite expanding supersymmetric open Freedman universe. Such a universe has the usual kind of asymptotic future consisting of time-like and light-like infinity. There is no temperature in the remote future and the geometry permits particles to separate and propagate as free particles just as in flat space-time.

Now let's consider the observer's past history. The same argument which says that the observer will eventually make a transition to  $\lambda = 0$  in the far future can be run backward. The observer could only have gotten to the de Sitter vacuum by the time-reversed history so he must have originated from a collapsing open universe. The entire history is shown in Figure 5. The history may seem paradoxical since it requires the second law of thermodynamics to be violated in the past. A similar paradox arises in a more familiar setting. Let me return to the sealed room filled with gas molecules except that now one of the walls has a small hole that lets the gas escape to unbounded space. Suppose we find the gas filling the room in thermal equilibrium at some time. If we run the system forward we will eventually find all the molecules have escaped and are on their way out, never to return. But it is also true that if we run the equations of motion backwards we will eventually find all the molecules outside the room moving away. Thus the only way the starting configuration could have occurred is if the original molecules were converging from infinity toward the small hole in the wall.

If we are studying the system quantum mechanically, the metastable configuration with all the molecules in the room would be an unstable *resonance* in a scattering matrix describing the many body scattering of a system of molecules with the walls of the room. Indeed the energy levels describing the molecules trapped inside the room are complex due to the finite lifetime of the configuration.

This suggests a view of the intermediate de Sitter Space in Figure 5 as an unstable resonance in the scattering matrix connecting states in the asymptotic  $\lambda = 0$  vacua. In fact we can estimate the width of the states. Since the lifetime of the de Sitter Space is always longer than the recurrence time, generally by a huge factor, the width  $\gamma$  satisfies

$$\gamma \gg \exp -S.$$

On the other hand the spacing between levels,  $\Delta E$ , is of order  $\exp -S$ . Therefore

$$\gamma \gg \Delta E \tag{5.1}$$



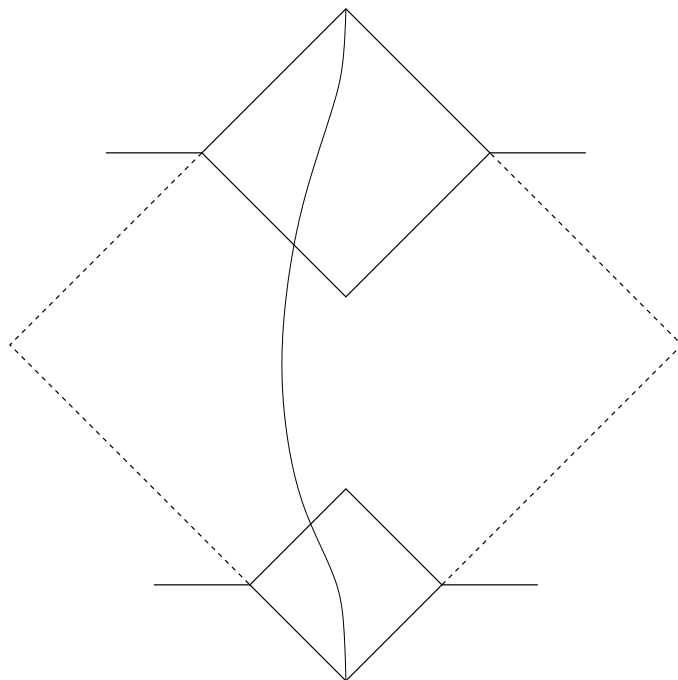


Figure 5:

so that the levels are very broad and overlapping as for the black hole.

No perfectly precise definition exists in string theory for the moduli fields or their potential when we go away from the supermoduli-space. The only precise definition of the de Sitter vacua seems to be as complex poles in some new sector of the scattering matrix between states on the supermoduli-space.

Knowing that a black hole is a resonance in a scattering amplitude does not tell us much about the way real black holes form. Most of the possibilities for black hole formation are just the time reverse of the ways that it evaporate. In other words the overwhelming number of initial states that can lead to a black hole consist of thermal radiation. Real black holes in our universe form from stellar collapse which is just one channel in a huge collection of S-matrix “in states”. In the same way the fact that cosmological states may be thought of in a scattering framework is in itself does not shed much light on the original creation process.

## 6 Conclusion

Vacua come in two varieties, supersymmetric and otherwise. Most likely the non-supersymmetric vacua do not have vanishing cosmological constant but it is plausible that there are so many of them that they practically form a continuum. Some tiny fraction have cosmological constant in the observed range. With nothing preferring one vacuum over another, the anthropic principle comes to the fore whether or not we like the idea. String theory provides a framework in which this can be studied in a rigorous way. Progress can certainly be made in exploring the landscape. The project is in its infancy but in time we should know just how rich it is. We can argue the philosophical merits of the anthropic principle but we can’t argue with quantitative information about the number of vacua with each particular property such as the cosmological constant, Higgs mass or fine structure constant. That information is there for us to extract.

Counting the vacua is important but not sufficient. More understanding of cosmological evolution is essential to determining if the large number of possibilities are realized as actualities. The vacua in string theory with  $\lambda > 0$  are not stable and decay on a time scale smaller than the recurrence time. This is very general and also very fortunate since there are serious problems with stable de Sitter space.

The instability also allows the universe to sample all or a large part of the landscape by means of bubble formation. In such a world the probability that some region of space

has suitable conditions for life to exist can be large.

The bubble universe based on Linde's eternal inflation seems promising but it is unclear how to think about it with precision. There are real conceptual problems having to do with the global view of spacetime. The main problem is to reconcile two pictures; the causal patch picture and the global picture. String theory has provided a testing ground for some important relevant ideas such as black hole complementarity [3, 4] and the Holographic principle [5, 6]. Complementarity requires the observer's side of the horizon to have a self contained conventional quantum description. It also prohibits a conventional quantum description that covers the interior and exterior simultaneously. Any attempt to describe both sides as a single quantum system will come into conflict with one of three sacred principles [13]. The first is the equivalence principle which says that a freely falling observer passes the horizon without incident. The second says that experiments performed outside a black hole should be consistent with the rules of quantum mechanics as set down by Dirac in his textbook. No loss of quantum information should take place and the time evolution should be unitary. Finally the rules of quantum mechanics forbid information duplication. This means that we can not resolve the so called information paradox by creating two copies (quantum Xeroxing) of every bit as it falls through the horizon; at least not within the formalism of conventional quantum mechanics. The complementarity and holographic principles have been convincingly confirmed by the modern methods of string theory [12]. The inevitable conclusion is that a global description of geometries with horizons, if it exists at all, will not be based on the standard quantum rules.

Why is this important for cosmology? The point is that the eternal inflationary production of an infinity of bubbles takes place behind the horizon of any given observer. It is not something that has a description within one causal patch. If it makes sense, a global description is needed but if cosmic event horizons are at all like black hole horizons then any global description will involve wholly new elements. If I were to make a wild guess about which rule of quantum mechanics has to be given up in a global description of either black holes or cosmology I would guess it is the Quantum Xerox Principle [13]. I would look for a theory which formally allowed quantum duplication but cleverly prevents any observer from witnessing it. Perhaps then the replication of bubbles can be sensibly described.

Progress may also be possible in sharpening the exact mathematical meaning of the de Sitter vacua. Away from the supermoduli-space, the concept of a local field and the effective potential is at best approximate in string theory. The fact that the vacua are

false metastable states makes it even more problematic to be precise. In ordinary quantum mechanics the best mathematical definition of an unstable state is as a resonance is amplitudes for scattering between very precisely defined asymptotic states. Each metastable state corresponds to pole whose real and imaginary parts define the energy and inverse lifetime of the state.

I have argued that each causal patch begins and ends with an asymptotic “roll” toward the supermoduli-space. The final state have the boundary conditions of an FRW open universe and the initial states are time reversals of these. This means we may be able to define some kind S-matrix connecting initial and final asymptotic states. The various intermediate metastable de Sitter phases would be exactly defined as resonant resonances in this amplitude.

At first this proposal sounds foolish. In general relativity initial and final states are very different. Black holes make sense. White holes do not. Ordinary things fall into black holes and thermal radiation comes out. The opposite never happens. But this is deceiving. Our experience with string theory has made it clear that the fundamental micro-physical input is completely reversible and that black holes are most rigorously defined in terms of resonances in scattering amplitudes <sup>6</sup>. Of course knowing that a black hole is an intermediate state in a tremendously complicated scattering amplitude does not really tell us much about how real black holes form. For that we need to know about stellar collapse and the like. But it does provide an exact mathematical definition of the states that comprise the black hole ensemble.

To further illustrate the point let me tell a story:

Two future astronauts in the deep empty reaches of outer space discover a sealed capsule. On further inspection they find a tiny pin-hole in the capsule and air is slowly leaking out. One says to the other, “Aha, we have discovered an eternal air tank. It must have been here forever.” The other says “No, you fool. if it were here forever the air would have leaked out long ago (infinitely long ago).” So the first one thinks and says, “Yes, you are right. Let’s think. If we wait long enough, all the air will be streaming outward in an asymptotic final state. That is clear. But, because of micro-reversibility, it is equally clear that if we go far into the past that all the air must have been doing the reverse. In fact the quantum states with air in the capsule are just intermediate resonances in the scattering of a collection of air molecules with the empty capsule.” The second astronaut looks at

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<sup>6</sup>The one exception is black holes in anti-de Sitter space which are stable.

him as if he were nuts. “Don’t be a dope. That’s just too unlikely. I guess someone else was here not so long ago and filled it up.”

Both of them can be right. The quantum states of air in a tank are mathematical resonances in a scattering matrix. And it may also be true that the laws of an isolated system of gas and tank may have been temporarily interfered with by another presence. Or we might say that the scattering states need to include not only air and tank but also cosmonauts and their apparatus.

It’s in this sense that I propose that de Sitter Space can be mathematically defined in terms of singularities in some kind of generalized S-matrix. But in so doing, I am not really telling you much about how it all started.

From the causal patch viewpoint the evolutionary endpoints seems to be an approach to some point on the supermoduli-space. After the last tunneling the universe enters an final open FRW expansion toward some flat supersymmetric solution. This is not to be thought of as a unique quantum state but as a large set of states with similar evolution. Running the argument backward (assuming microscopic reversibility) we expect the initial state to be the time reversal of one of the many future endpoints. We might even hope for a scattering matrix connecting initial and final states. de Sitter minima would be an enormously large density of complex poles in the amplitude.

One last point: The final and initial states do not have to be four dimensional. In fact in the example given in [7], the modulus describing the overall size of the compact space roles to infinity, thus creating a ten or possibly and eleven dimensional universe.

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